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CHARACTERIZATIONS OF HIGHER DERIVATIONS

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Abstract

Let $\mathcal{A}$ be a unital algebra and $D = (\delta_i)_{i\in\mathbb{N}}$ be a family of linear mappings from $\mathcal{A}$ into itself such that $\delta_0 = id_{\mathcal{A}}$. In this paper, we prove that if $\sum_{i+j+k=n}\delta_i(A)\delta_j(B)\delta_k(C) = 0$ for any $A, B, C \in \mathcal{A}$ with $AB = BC = 0$ and $\delta_n(I) = 0$ for all $n \geq 1$, then the restriction $D = (\delta_i)_{i\in\mathbb{N}}$ to the subalgebra $\mathcal{R}$ generated by all idempotents of $\mathcal{A}$ is a higher derivation. We also show that this kind of mappings is a higher derivation on $\mathcal{A}$ under some conditions.

1. Introduction

Let $\mathcal{A}$ be a unital algebra. Let $D = (\delta_i)_{i\in\mathbb{N}}$ be a family of linear mappings from $\mathcal{A}$ into itself such that $\delta_0 = id_{\mathcal{A}}$. D is called a *higher derivation*, if $\delta_n(AB) = \sum_{i+j=n}\delta_i(A)\delta_j(B)$ for each $n \in \mathbb{N}$ and $A, B \in \mathcal{A}$; D is called a *Jordan higher derivation*, if $\delta_n(A^2) = \sum_{i+j=n}\delta_i(A)\delta_j(A)$ for each $n \in \mathbb{N}$ and $A \in \mathcal{A}$. Note that δ_1

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is a Jordan derivation, if $D = (\delta_i)_{i\in\mathbb{N}}$ is a Jordan higher derivation. It is well known that every derivation is a Jordan derivation and the converse in general is not true. In [8], Herstein showed that every Jordan derivation from a 2-torsion free prime ring into itself is a derivation. In [2], Brešar generalized Herstein's result to 2-torsion free semiprime rings. Likewise, every higher derivation is a Jordan higher derivation and the converse in general is not true. In [6], Ferrero and Haetinger generalized Brešar's result to the Jordan higher derivations, they showed that every Jordan higher derivation of a 2-torsion free semiprime ring is a higher derivation. For other related results, see [14, 15].

In general, there are two directions in the study of the local actions of derivations of operator algebras. One is the local derivation problem (for example, see [5, 10, 11]). The other is to study the conditions under which derivations of operator algebras can be completely determined by the action on some sets of operators (for example, see [1, 4, 9, 12]). In [3], Brešar study the local actions of derivations. He proved that if δ is an additive mapping from a unital ring $\mathcal{A}$ to a unital $\mathcal{A}$-bimodule $\mathcal{M}$ such that $A\delta(B)C = 0$ for all $AB = BC = 0$, then the restriction δ to the subring $\mathcal{R}$ generated by all idempotents of $\mathcal{A}$ is a derivation. In [12], Li and Pan showed that under some conditions, this kind of mappings is a generalized derivation on a unital algebra $\mathcal{A}$. In Section 2, we generalize Brešar's result and Li's result to the case of higher derivations, respectively.

2. Main Results

In this section, we always assume that $\mathcal{A}$ is a unital algebra.

Let $D = (\delta_i)_{i\in\mathbb{N}}$ be a family of linear mappings from $\mathcal{A}$ into itself. We say that $D = (\delta_i)_{i\in\mathbb{N}}$ satisfies the condition $(*)$ if for each $A \in \mathcal{A}$, any idempotents $P, Q \in \mathcal{A}$ and all $n \in \mathbb{N}$;

$$\delta_n(PAQ) = \sum_{i+j=n} \delta_i(PA)\delta_j(Q) + \sum_{i+j=n} \delta_i(P)\delta_j(AQ)$$

$$-\sum_{i+j+k=n} \delta_i(P)\,\delta_j(A)\,\delta_k(Q), \tag{$*$}$$

and

$$\delta_n(I) = 0, \qquad \text{for all } n \geq 1.$$

In order to prove our main result, we first show two lemmas.

Lemma 2.1. *Suppose that* $D = (\delta_i)_{i\in\mathbb{N}}$ *is a family of linear mappings from* $\mathcal{A}$ *into itself satisfying condition* $(*)$. *Then for any idempotents* $P_1, \cdots, P_m$ *in* $\mathcal{A}$,

(1) $\delta_n(P_1 \cdots P_m) = \sum_{i+j=n} \delta_i(P_1)\,\delta_j(P_2 \cdots P_m)$,

(2) $\delta_n(P_1 \cdots P_m) = \sum_{i+j=n} \delta_i(P_1 \cdots P_{m-1})\,\delta_j(P_m)$.

Proof. We only prove (1), for the proof of (2) is analogous.

When $m = 1, 2,$ by the condition $(*)$, (1) is true. Suppose that if $m = t$, (1) is true.

For $m = t+1$, by the condition $(*)$, it follows that

$$\begin{aligned}\delta_n(P_1 \cdots P_{t+1}) &= \sum_{i+j=n} \delta_i(P_1 \cdots P_t)\,\delta_j(P_{t+1}) + \sum_{i+j=n} \delta_i(P_1)\,\delta_j(P_2 \cdots P_t P_{t+1}) \\ &\quad - \sum_{i+j+k=n} \delta_i(P_1)\,\delta_j(P_2 \cdots P_t)\,\delta_k(P_{t+1}) \\ &= \sum_{i+j=n} \delta_i(P_1)\,\delta_j(P_2 \cdots P_t P_{t+1}).\end{aligned}$$

This concludes the proof. □

Lemma 2.2. *Suppose that* $D = (\delta_i)_{i\in\mathbb{N}}$ *is a family of linear mappings from* $\mathcal{A}$ *into itself satisfying condition* $(*)$. *Then for any idempotents* $P_1, \cdots, P_m, Q_1, \cdots, Q_s$ *in* $\mathcal{A}$ *and every* $A \in \mathcal{A}$,

$$\delta_n(P_1 \cdots P_m A Q_1 \cdots Q_s)$$

$$= \sum_{i+j=n} \delta_i(P_1 \cdots P_m A)\delta_j(Q_1 \cdots Q_s) + \sum_{i+j=n} \delta_i(P_1 \cdots P_m)\delta_j(AQ_1 \cdots Q_s)$$

$$- \sum_{i+j+k=n} \delta_i(P_1 \cdots P_m)\delta_j(A)\delta_k(Q_1 \cdots Q_s). \tag{2.1}$$

Proof. We first show that for any positive integer m,

$$\delta_n(P_1 \cdots P_m AQ) = \sum_{i+j=n} \delta_i(P_1 \cdots P_m A)\delta_j(Q) + \sum_{i+j=n} \delta_i(P_1 \cdots P_m)\delta_j(AQ)$$

$$- \sum_{i+j+k=n} \delta_i(P_1 \cdots P_m)\delta_j(A)\delta_k(Q). \tag{2.2}$$

If $m = 1$, by the condition $(*)$, (2.2) is true. Suppose that if $m = t$, (2.2) is true.

For $m = t + 1$, by the condition $(*)$ and Lemma 2.1, it follows that

$$\delta_n(P_1 \cdots P_{t+1} AQ)$$

$$= \sum_{i+j=n} \delta_i(P_1 \cdots P_{t+1} A)\delta_j(Q) + \sum_{i+j=n} \delta_i(P_1)\delta_j(P_2 \cdots P_{t+1} AQ)$$

$$- \sum_{i+j+k=n} \delta_i(P_1)\delta_j(P_2 \cdots P_{t+1} A)\delta_k(Q)$$

$$= \sum_{i+j=n} \delta_i(P_1 \cdots P_{t+1} A)\delta_j(Q) + \sum_{i+j+k=n} \delta_i(P_1)\delta_j(P_2 \cdots P_{t+1})\delta_k(AQ)$$

$$- \sum_{i+j+k+l=n} \delta_i(P_1)\delta_j(P_2 \cdots P_{t+1})\delta_k(A)\delta_l(Q)$$

$$= \sum_{i+j=n} \delta_i(P_1 \cdots P_{t+1} A)\delta_j(Q) + \sum_{i+j=n} \delta_i(P_1 P_2 \cdots P_{t+1})\delta_j(AQ)$$

$$- \sum_{i+j+k=n} \delta_i(P_1 P_2 \cdots P_{t+1})\delta_j(A)\delta_k(Q).$$

Now, we show (2.1) is true.

If $s = 1,$ by (2.2), (2.1) is true. Suppose that if $s = t,$ (2.1) is true.

For $s = t + 1,$ by (2.2), the condition $(*)$ and Lemma 2.1, it follows that

$$\begin{aligned}
&\delta_n(P_1 \cdots P_m A Q_1 \cdots Q_{t+1}) \\
&= \sum_{i+j=n} \delta_i(P_1 \cdots P_m A Q_1 \cdots Q_t)\delta_j(Q_{t+1}) + \sum_{i+j=n} \delta_i(P_1 P_2 \cdots P_m)\delta_j(A Q_1 \cdots Q_{t+1}) \\
&\quad - \sum_{i+j+k=n} \delta_i(P_1 P_2 \cdots P_m)\delta_j(A Q_1 \cdots Q_t)\delta_k(Q_{t+1}) \\
&= \sum_{i+j+k=n} \delta_i(P_1 \cdots P_m A)\delta_j(Q_1 \cdots Q_t)\delta_k(Q_{t+1}) \\
&\quad + \sum_{i+j=n} \delta_i(P_1 P_2 \cdots P_m)\delta_j(A Q_1 \cdots Q_{t+1}) \\
&\quad - \sum_{i+j+k+l=n} \delta_i(P_1 \cdots P_m)\delta_j(A)\delta_k(Q_1 \cdots Q_t)\delta_l(Q_{t+1}) \\
&= \sum_{i+j=n} \delta_i(P_1 \cdots P_m A)\delta_j(Q_1 \cdots Q_{t+1}) + \sum_{i+j=n} \delta_i(P_1 P_2 \cdots P_m)\delta_j(A Q_1 \cdots Q_{t+1}) \\
&\quad - \sum_{i+j+k=n} \delta_i(P_1 \cdots P_m)\delta_j(A)\delta_k(Q_1 \cdots Q_{t+1}).
\end{aligned}$$

This concludes the proof. □

Theorem 2.3. *Let $\mathcal{A}$ be a unital algebra and $\mathcal{B}$ be the subalgebra generated by all idempotents in $\mathcal{A}$. If $D = (\delta_i)_{i\in\mathbb{N}}$ is a family of linear mappings from $\mathcal{A}$ into itself such that $\delta_n(ABC) = \sum_{i+j+k=n}\delta_i(A)\delta_j(B)\delta_k(C)$ for any $A, B, C \in \mathcal{A}$ with $AB = BC = 0$ and $\delta_n(I) = 0$ for all $n \geq 1,$ then the restriction of $D = (\delta_i)_{i\in\mathbb{N}}$ to $\mathcal{B}$ is a higher derivation.*

Proof. Let P and Q be two idempotents in $\mathcal{A}$. Since for every $A \in \mathcal{A}$,

$$(I-P)PAQ = PAQ(I-Q) = 0,$$

$$P(I-P)AQ = (I-P)AQ(I-Q) = 0,$$

$$(I-P)PA(I-Q) = PA(I-Q)Q = 0,$$

$$P(I-P)A(I-Q) = (I-P)A(I-Q)Q = 0,$$

we have

$$\sum_{i+j+k=n} \delta_i(I-P)\,\delta_j(PAQ)\,\delta_k(I-Q) = 0,$$

$$\sum_{i+j+k=n} \delta_i(P)\,\delta_j((I-P)AQ)\,\delta_k(I-Q) = 0,$$

$$\sum_{i+j+k=n} \delta_i(I-P)\,\delta_j(PA(I-Q))\,\delta_k(Q) = 0,$$

$$\sum_{i+j+k=n} \delta_i(P)\,\delta_j((I-P)A(I-Q))\,\delta_k(Q) = 0.$$

For convenience, we rewrite these identities as

$$\delta_n(PAQ) = \sum_{i+j=n} \delta_i(PAQ)\,\delta_j(Q) + \sum_{i+j=n} \delta_i(P)\,\delta_j(PAQ) - \sum_{i+j+k=n} \delta_i(P)\,\delta_j(PAQ)\,\delta_k(Q),$$

$$-\sum_{i+j=n} \delta_i(P)\,\delta_j(AQ) = -\sum_{i+j=n} \delta_i(P)\,\delta_j(PAQ) - \sum_{i+j+k=n} \delta_i(P)\,\delta_j(AQ)\,\delta_k(Q) + \sum_{i+j+k=n} \delta_i(P)\,\delta_j(PAQ)\,\delta_k(Q),$$

$$-\sum_{i+j=n} \delta_i(PA)\,\delta_j(Q) = -\sum_{i+j=n} \delta_i(PAQ)\,\delta_j(Q)$$

$$- \sum_{i+j+k=n} \delta_i(P)\,\delta_j(PA)\,\delta_k(Q)$$

$$+ \sum_{i+j+k=n} \delta_i(P)\,\delta_j(PAQ)\,\delta_k(Q),$$

$$\sum_{i+j+k=n} \delta_i(P)\delta_j(A)\delta_k(Q) = \sum_{i+j+k=n} \delta_i(P)\delta_j(PA)\delta_k(Q)$$

$$+ \sum_{i+j+k=n} \delta_i(P)\delta_j(AQ)\delta_k(Q)$$

$$- \sum_{i+j+k=n} \delta_i(P)\delta_j(PAQ)\delta_k(Q).$$

Note that the sum of the right-hand sides of these four identities is 0. Therefore, the sum of the left-hand sides must be 0. Hence,

$$\delta_n(PAQ) = \sum_{i+j=n} \delta_i(P)\,\delta_j(AQ) + \sum_{i+j=n} \delta_i(PA)\,\delta_j(Q)$$

$$- \sum_{i+j+k=n} \delta_i(P)\,\delta_j(A)\,\delta_k(Q).$$

By Lemma 2.2, we have for any idempotents $P_1, \cdots, P_m, Q_1, \cdots, Q_s$ in $\mathcal{A}$ and every $A \in \mathcal{A}$,

$$\delta_n(P_1 \cdots P_m A Q_1 \cdots Q_s)$$

$$= \sum_{i+j=n} \delta_i(P_1 \cdots P_m A)\,\delta_j(Q_1 \cdots Q_s) + \sum_{i+j=n} \delta_i(P_1 \cdots P_m)\,\delta_j(AQ_1 \cdots Q_s)$$

$$- \sum_{i+j+k=n} \delta_i(P_1 \cdots P_m)\delta_j(A)\delta_k(Q_1 \cdots Q_s).$$

Setting $A = I$ in the above relation, we obtain

$$\delta_n(P_1 \cdots P_m Q_1 \cdots Q_s) = \sum_{i+j=n} \delta_i(P_1 \cdots P_m)\delta_j(Q_1 \cdots Q_s).$$

This concludes the proof. $\square$

Let $\mathcal{M}$ be an $\mathcal{A}$-module and $\mathcal{J}$ be an ideal of $\mathcal{A}$. We say that $\mathcal{J}$ is a *separating set* of $\mathcal{M}$, if for every $m, n \in \mathcal{M}$, $m\mathcal{J} = 0$ implies $m = 0$ and $\mathcal{J}n = 0$ implies $n = 0$.

Theorem 2.4. *Let $\mathcal{J}$ be a separating set of $\mathcal{A}$. Suppose $\mathcal{J}$ is contained in the linear span of the idempotents in $\mathcal{A}$. If $D = (\delta_i)_{i\in\mathbb{N}}$ is a family of linear mappings from $\mathcal{A}$ into itself satisfying condition $(*)$, then $D = (\delta_i)_{i\in\mathbb{N}}$ is a higher derivation.*

Proof. When $n = 1$, by [7, Theorem 2.2], we have δ_1 is a derivation. Now we assume that

$$\delta_m(AB) = \sum_{i+j=m} \delta_i(A)\delta_j(B),$$

for all $A, B \in \mathcal{A}$ and for all $1 \le m < n$.

Since $\mathcal{J}$ is contained in the linear span of the idempotents in $\mathcal{A}$, by the condition $(*)$, it follows that for any $S, T \in \mathcal{J}$,

$$\delta_n(ST) = \sum_{i+j=n} \delta_i(S)\delta_j(T).$$

For any $S, T \in \mathcal{J}$ and $A \in \mathcal{A}$. Since $\mathcal{J}$ is an ideal of $\mathcal{A}$, it follows that

$$\delta_n(SAT) = \delta_n((SA)T) = \sum_{i+j=n} \delta_i(SA)\delta_j(T). \tag{2.3}$$

On the other hand, by the condition $(*)$,

$$\delta_n(SAT) = \sum_{i+j=n} \delta_i(SA)\delta_j(T) + \sum_{i+j=n} \delta_i(S)\delta_j(AT)$$
$$- \sum_{i+j+k=n} \delta_i(S)\delta_j(A)\delta_k(T). \tag{2.4}$$

Combining (2.3) and (2.4), we have

$$0 = \sum_{i+j=n} \delta_i(S)\delta_j(AT) - \sum_{i+j+k=n} \delta_i(S)\delta_j(A)\delta_k(T)$$

$$= S\delta_n(AT) - S\sum_{i+j=n}\delta_i(A)\,\delta_j(T).$$

Since $\mathcal{J}$ is a separating set of $\mathcal{A}$, it follows that

$$\delta_n(AT) = \sum_{i+j=n}\delta_i(A)\,\delta_j(T).$$

For any $A, B \in \mathcal{A}$ and for any $T \in \mathcal{J}$, we have

$$\delta_n(ABT) = \sum_{i+j=n}\delta_i(A)\delta_j(BT) = \sum_{i+j+k=n}\delta_i(A)\delta_j(B)\delta_k(T)$$

$$= \sum_{i+j=n}\delta_i(A)\delta_j(B)T + \sum_{\substack{i+j=n\\ j\geq 1}}\delta_i(AB)\delta_j(T).$$

On the other hand,

$$\delta_n(ABT) = \sum_{i+j=n}\delta_i(AB)\delta_j(T) = \delta_n(AB)T + \sum_{\substack{i+j=n\\ j\geq 1}}\delta_i(AB)\,\delta_j(T).$$

So, we have

$$\delta_n(AB)T - \sum_{i+j=n}\delta_i(A)\delta_j(B)T = 0.$$

Since $\mathcal{J}$ is a separating set of $\mathcal{A}$, it follows that

$$\delta_n(AB) = \sum_{i+j=n}\delta_i(A)\delta_j(B).$$

Hence, $D = (\delta_i)_{i\in\mathbb{N}}$ is a higher derivation. $\square$

A linear mapping f from $\mathcal{A}$ to $\mathcal{M}$ is called a *left* (*resp*., *right*) *multiplier*, if $f(a) = f(1)a$ (resp., $f(a) = af(1)$), for every $a \in \mathcal{A}$. Clearly, left multipliers are left-annihilator-preserving and right multipliers are right-annihilator-preserving. With certain hypotheses on $\mathcal{A}$ and $\mathcal{M}$, multipliers are the only annihilator-preserving maps from $\mathcal{A}$ to $\mathcal{M}$ (see [12]). When this happens, we have the following theorem:

Theorem 2.5. *Let $\mathcal{A}$ be a unital algebra. Suppose that the only linear left-annihilator-preserving maps from $\mathcal{A}$ into itself are left multipliers and the only linear right-annihilator-preserving maps from $\mathcal{A}$ into itself are right multipliers. If $D = (\delta_i)_{i\in\mathbb{N}}$ is a family of linear mappings from $\mathcal{A}$ into itself such that $\sum_{i+j+k=n} \delta_i(A)\delta_j(B)\delta_k(C) = 0$ for any $A, B, C \in \mathcal{A}$ with $AB = BC = 0$ and $\delta_n = (I) = 0$ for all $n \geq 1$, then $D = (\delta_i)_{i\in\mathbb{N}}$ is a higher derivation.*

Proof. When $n = 1$, $A\delta_1(B)C = 0$ for any $A, B, C \in \mathcal{A}$ with $AB = BC = 0$. By [12, Proposition 1.1], we have δ_1 is a derivation. Now we assume that

$$\delta_m(ST) = \sum_{i+j=m} \delta_i(S)\delta_j(T),$$

for all $S, T \in \mathcal{A}$ and for all $1 \leq m < n$. Then

$$\sum_{i+j+k=n} \delta_i(A)\delta_j(B)\delta_k(C) = \sum_{\substack{i+j+k=n\\ k\geq 1}} \delta_i(A)\delta_j(B)\delta_k(C) + \sum_{i+j=n} \delta_i(A)\delta_j(B)C$$

$$= \sum_{i+j=n} \delta_i(A)\delta_j(B)C.$$

So $\sum_{i+j=n}\delta_i(A)\delta_j(B)C = 0$. Fix $A, B \in \mathcal{A}$ with $AB = 0$, define a mapping f depending on A and B from $\mathcal{A}$ into itself by

$$f(T) = \sum_{i+j=n} \delta_i(A)\delta_j(BT),$$

for any $T \in \mathcal{A}$. For any $C, D \in A$ with $CD = 0$, we have $ABC = BCD = 0$. So $f(C)D = 0$. By the hypotheses, f is a left multiplier, that is, $f(S) = f(I)S$ for any $S \in A$. Thus,

$$0 = \sum_{i+j=n} \delta_i(A)\,\delta_j(BS) - \sum_{i+j=n} \delta_i(A)\,\delta_j(B)S$$

$$= A\delta_n(BS) + \sum_{\substack{i+j+k=n \\ i\geq 1}} \delta_i(A)\,\delta_j(B)\,\delta_k(S) - \sum_{i+j=n} \delta_i(A)\,\delta_j(B)S$$

$$= A\delta_n(BS) + \sum_{\substack{i+j+k=n \\ i\geq 1,\, k\geq 1}} \delta_i(A)\,\delta_j(B)\,\delta_k(S) + \sum_{\substack{i+j=n \\ i\geq 1}} \delta_i(A)\,\delta_j(B)S$$

$$- \sum_{i+j=n} \delta_i(A)\,\delta_j(B)S$$

$$= A\delta_n(BS) - A\sum_{\substack{i+j=n \\ j\geq 1}} \delta_i(B)\,\delta_j(S) - A\delta_n(B)S$$

$$= A\delta_n(BS) - A\sum_{i+j=n} \delta_i(B)\,\delta_j(S).$$

Let

$$g(T) = \delta_n(TS) - \sum_{i+j=n} \delta_i(T)\,\delta_j(S).$$

Then $Ag(B) = 0$. By the hypotheses, g is a right multiplier, that is, $g(T) = Tg(1)$ for any $T \in A$. Since $g(I) = 0$, we have $g(T) = 0$. Thus,

$$\delta_n(TS) = \sum_{i+j=n} \delta_i(T)\delta_j(S).$$

Hence, $D = (\delta_i)_{i\in\mathbb{N}}$ is a higher derivation. $\square$

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